

## pair\_support\_double\_sum<sup>4,23</sup>

$\forall n, m: \mathbb{N}, f: (\mathbb{N}_{<n} \rightarrow \mathbb{N}_{<m} \rightarrow \mathbb{Z}), x_1, x_2: \mathbb{N}_{<n}, y_1, y_2: \mathbb{N}_{<m}.$

$\neg x_1 = x_2 \in \mathbb{Z} \vee \neg y_1 = y_2 \in \mathbb{Z}$

$\Rightarrow (\forall x: \mathbb{N}_{<n}, y: \mathbb{N}_{<m}. \neg(x = x_1 \in \mathbb{Z} \ \& \ y = y_1 \in \mathbb{Z}) \Rightarrow \neg(x = x_2 \in \mathbb{Z} \ \& \ y = y_2 \in \mathbb{Z}) \Rightarrow f(x, y) = 0)$

$\Rightarrow \text{sum}(f(x, y) \mid x < n; y < m) = f(x_1, y_1) + f(x_2, y_2)$